

CBCS SCHEME

18MAT41



Fourth Semester B.E. Degree Examination, Jan./Feb. 2021

Complex Analysis, Probability and Statistical Methods

Time: 3-hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Show that $W = \log Z$, $Z \neq 0$ is analytic and hence find $\frac{dw}{dz}$. (06 Marks)
- b. Derive Cauchy – Riemann equation in Cartesian coordinates. (07 Marks)
- c. Find the analytic function $f(z) = u + iv$ given $v = e^{-x} [x \cos y + y \sin y]$. (07 Marks)

OR

- 2 a. Show that an analytic function with constant modulus is constant. (06 Marks)
- b. If $f(z) = u + iv$ is analytic prove that

$$\left[\frac{\partial f(z)}{\partial x} \right]^2 + \left[\frac{\partial f(z)}{\partial y} \right]^2 = |f'(z)|^2$$
 (07 Marks)
- c. If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is analytic function, if $z = x + iy$, find $f(z)$ in terms of z . (07 Marks)

Module-2

- 3 a. State and prove Cauchy's Integral formula. (06 Marks)
- b. Discuss the transformation $W = e^z$. (07 Marks)
- c. Find the Bilinear transformation which sends points $Z = 0, 1, \infty$ into the points $W = -5, -1, 3$ respectively. What are the invariant points in this transformation? (07 Marks)

OR

- 4 a. Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the line i) $y = x$ ii) $y = x^2$. (06 Marks)
- b. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. (07 Marks)
- c. Find the Bilinear transformation that maps the points $Z = -1, i, 1$ onto the points $W = 1, i, -1$. Also find the invariant points. (07 Marks)

Module-3

- 5 a. A random variable X has the following probability function :

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

- Find i) the value of k ii) $P(x < 6)$ iii) $P(x \geq 6)$ iv) $P(3 < x \leq 6)$. (06 Marks)
- b. The probability that a pen manufactured by a company be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that i) Exactly 2 are defective ii) at least 2 are defective iii) none of them are defective. (07 Marks)
 - c. A sample of 100 battery cells tested to find the length of life produced the following results $\bar{x} = 12$ hours, $\sigma = 3$ hours. Assuming the data to be normally distributed what percentage of battery cells are expected to have life. i) more than 15 hours ii) less than 6 hours iii) between 10 and 14 hours. [$A(1) = 0.3413$, $A(2) = 0.4772$, $A(0.67) = 0.2487$]. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



6 a. The probability density function of a random variable ($y = x$) is

$$f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

b. A certain screw machine produces on average two defective out of 100 and packs them in boxes of 500. Find the probability that the box contains

- i) At least one defective
 - ii) between 2 and 4 defective.
 - iii) between 2 and 4 defective.
- c. The length of a telephone conversation is an exponential variate with mean 3 minute. Find the probability that a call
- i) ends in less than 3 minutes
 - ii) between 3 and 5 minutes.

(07 Marks)

OR

7 a. Ten students get the following percentage of marks in two subjects A and B.

Marks in A	78	36	98	25	75	82	90	62	65	39
Marks in B	84	51	91	60	68	62	86	58	53	47

(06 Marks)

b. Fit a best fitting parabola $Y = a + bx + cx^2$ for the following data :

x	-2	-1	0	1	2
y	-3.150	-1.390	0.620	2.880	5.378

(07 Marks)

c. The regression lines $4x - 5y + 3z = 0$ and $20x - 9y = 107$. Find

- i) the mean values of x and y
- ii) the correlation between x and y
- iii) the variance of Y given that the variance of X is 9.

(07 Marks)

OR

8 a. If θ is the acute angle between the lines of regression then show that

$$\tan \theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{r^2 - 1}{r} \right)$$

(06 Marks)

b. Find the coefficient of correlation and the equation of regression lines for the following data

x	1	2	3	4	5	6	7	8	9
y	12	11	13	15	14	17	16	19	18

(07 Marks)

c. Fit a best fitting curve in the form $y = ax^b$ for the following data :

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(07 Marks)

9 a. The joint probability distribution for two random variables X and Y is as given below :

X \ Y	2	1	0
2	0.2	0.1	0.1
1	0.1	0.2	0
0	0.3	0	0

b. Find the marginal distribution of X and Y. Also find the Covariance of X and Y. (06 Marks)

In 324 throws of a six faced die an odd number turned up 181 times. It is reasonable to think that the die is an unbiased at 0.01 level of significance. (07 Marks)

- c. The nine item of a sample have the following values 45 , 47 , 50 , 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? (07 Marks)

OR

- 10 a. Define the following terms :

- i) Null hypothesis ii) Type – I and Type II error iii) Level of significance. (06 Marks)
- b. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (Given $t_{0.05} = 2.262$ for gdf). (07 Marks)
- c. The theory predicts the proportion of beans in the four groups G1, G2, G3, G4 should be in the ratio 9:3:3:1. In an experiment with 1600 beans the numbers in the four group were 882 , 313 , 287 and 118. Does the experimental result support the theory (at 5% level of significance for 3)? (07 Marks)


